ABHYASMITRA.IN

STUDY NOTES

ARTIFICIAL INTELLIGENCE

Unit IV — Knowledge Representation using Logical Formalisms

Topics Covered (09 Hours)

Introduction to Logical Formalisms & Role of Logic in AI
Knowledge-based Agents (KB, Ask, and Tell Operations)
Syntax and Semantics of Logical Systems (Truth & Entailment)
Propositional Logic (Symbols, Formulas & Connectives)
Propositional Inference Rules (Modus Ponens & Modus Tollens)
Resolution in Propositional Logic (Conjunctive Normal Form)
First-Order Predicate Logic (FOL) Fundamentals
FOL Quantifiers (Universal ∀ and Existential ∃)
Translating Natural Language Statements into FOL Formulas
Inference in FOL (Instantiation, Unification & Resolution)

Compiled in a simple, easy-to-understand, book style format
for students of Computer Science and Engineering

TABLE OF CONTENTS

4.1 Introduction to Logical Formalisms

In Artificial Intelligence, representation is just as important as search. An agent must possess a representation of the world in order to reason about it, plan actions, and achieve goals. Logical Formalisms represent knowledge by translating facts about the real world into formal mathematical sentences. Logic has a key role in AI because it provides a mathematically rigorous language that enables unambiguous expression and sound, deductive reasoning.

Knowledge-Based Agents

A Knowledge-Based Agent is an agent whose internal state is structured as a collection of sentences in a formal representation language. This collection of sentences is called a Knowledge Base (KB). The agent interacts with the KB using two primary operations:

- **TELL:** Adds new sentences (facts or rules) to the Knowledge Base, representing new information learned from perception or instruction.
- **ASK:** Queries the Knowledge Base to determine what action to take or whether a particular statement is true, based on the current knowledge.

Syntax and Semantics of Logical Systems

Every logical system consists of two distinct components that define how the language is written and what it means:

- **Syntax:** The grammar of the language. It defines the formal rules for combining symbols to create well-formed sentences. For example, in arithmetic, 'x + y = 4' is syntactically correct, while '+ x 4 =' is not.
- **Semantics:** The meaning of the sentences. It defines the truth of a sentence with respect to a particular model (a possible world). If a model m assigns truth values to variables such that a sentence alpha is true, we say the model satisfies alpha.

Logical Entailment ($KB \models \alpha$)

Entailment is the relationship between sentences where one sentence follows logically from another. We write $KB \models \alpha$ to state that the knowledge base KB entails the sentence α.

Formally, $KB \models \alpha$ if and only if in every model where KB is true, α is also true.

4.2 Propositional Logic – Basics and Inference

Propositional Logic is the simplest logic system. It represents facts using propositional symbols (variables like P, Q, R) that represent complete sentences (e.g., P = 'It is raining', Q = 'The grass is wet').

Propositional Symbols and Well-Formed Formulas

A Well-Formed Formula (WFF) is a syntactically correct sentence constructed from propositional symbols and logical connectives. The rules are recursive: any proposition symbol is a WFF, and if A and B are WFFs, then statements joined by connectives are also WFFs.

The Logical Connectives and Truth Table

We combine propositional symbols using five basic logical connectives. Below is the complete truth table defining their semantics:

P	Q	Negation $\neg P$	Conjunction $P \wedge Q$	Disjunction $P \vee Q$	Implication $P \Rightarrow Q$	Biconditional $P \Leftrightarrow Q$
True	True	False	True	True	True	True
True	False	False	False	True	False	False
False	True	True	False	True	True	False
False	False	True	False	False	True	True

NOTE
The implication connective ($P \Rightarrow Q$, read as 'P implies Q') is only False when P is True and Q is False. If P is False, the implication is automatically True, regardless of the truth value of Q. This is known as a material implication.

Inference Rules in Propositional Logic

Logical inference is the process of proving new sentences from existing ones in the Knowledge Base (KB $\vdash \alpha$). We use sound inference rules to derive these new facts mechanically without constructing entire truth tables.

1. Modus Ponens (Implication Elimination)

If the sentences P and $P \Rightarrow Q$ are in the Knowledge Base, we can infer Q. This is the most fundamental rule of deduction.

Form: $[\, P \,,\, P \Rightarrow Q \,] \vdash Q$

2. Modus Tollens

If we know that $P \Rightarrow Q$ is true, and we know that Q is false ($\neg Q$), we can infer that P must also be false ($\neg P$).

Form: $[\, \neg Q \,,\, P \Rightarrow Q \,] \vdash \neg P$

3. Resolution (Unit Resolution)

Resolution is a powerful inference rule that forms the basis of automated theorem proving in modern AI systems. It operates on clauses (disjunctions of literals). If we have a clause containing literal A, and another clause containing the negation of A ($\neg A$), we can resolve them by taking the union of the remaining literals.

Form: $[\, A \lor B \,,\, \neg A \lor C \,] \vdash B \lor C$

To use resolution, the sentences in the Knowledge Base must first be converted into Conjunctive Normal Form (CNF)—a conjunction of disjunctions (e.g., $(A \lor B) \land (\neg A \lor C)$).

4.4 First Order Predicate Logic – Fundamentals

While Propositional Logic is simple and precise, it is too weak to represent complex real-world knowledge efficiently. It treats the world as a set of facts, lacking the expressiveness to talk about individual objects, properties of those objects, or relations between them. First-Order Logic (FOL) addresses this by introducing objects, relations, and quantifiers.

Components of First-Order Logic

- **Objects:** Concrete entities in the domain (e.g., John, Australia, Red, Disk1).
- **Relations (Predicates):** Properties of objects or relations between them (e.g., Red(Australia) represents a property, Brother(John, Richard) represents a relation).
- **Functions:** Mappings that return an object from an input (e.g., FatherOf(John) returns John's father).
- **Variables:** Placeholders that stand for any object in the domain (e.g., x, y).

Quantifiers

Quantifiers allow us to make statements about entire collections of objects rather than naming each object individually.

1. Universal Quantifier (∀ - 'For All')

A universal quantifier states that a predicate is true for every possible object in the domain.

$\forall x$ King(x) $\Rightarrow$ Person(x) (All kings are people)

2. Existential Quantifier (∃ - 'There Exists')

An existential quantifier states that a predicate is true for at least one object in the domain.

$\exists x$ King(x) $\land$ Greedy(x) (There exists a king who is greedy)

4.5 Translating Natural Language into FOL

Translating English sentences into First-Order Logic is a critical skill for building knowledge representation bases. It requires identifying the objects, variables, and relations correctly.

Common Sentence Translations

Natural Language Sentence	First-Order Logic Translation
Every student is smart.	$\forall x\ Student(x) \Rightarrow Smart(x)$
Some students are smart.	$\exists x\ Student(x) \land Smart(x)$
John has a brother who is a doctor.	$\exists x\ Brother(x, John) \land Doctor(x)$
No student is lazy.	$\forall x\ Student(x) \Rightarrow \neg Lazy(x)$
Only doctors are smart.	$\forall x\ Smart(x) \Rightarrow Doctor(x)$

Common Semantic Pitfalls

A very common error is mixing up the connectives associated with quantifiers:

- **Pitfall 1 (Using $\land$ with $\forall$):** Writing '$\forall x\ Student(x) \land Smart(x)$' does NOT mean 'All students are smart'. It means 'Everything in the universe is a student and is smart' (which is false, since cars and chairs are not students). Always use implication ($\Rightarrow$) with universal quantifiers ($\forall$).
- **Pitfall 2 (Using $\Rightarrow$ with $\exists$):** Writing '$\exists x\ Student(x) \Rightarrow Smart(x)$' is technically true even if there are no smart students, because the implication is true if the antecedent is false (e.g., if x is a chair, it is not a student, making $Student(chair) \Rightarrow Smart(chair)$ vacuously true). Always use conjunction ($\land$) with existential quantifiers ($\exists$).

4.6 Inference in First Order Predicate Logic

Inference in First-Order Logic is more complex than in Propositional Logic because we must handle variables and quantifiers. We do this by either instantiating variables (grounding them) or using algorithms like unification to compare variables directly.

1. Quantifier Instantiation Rules

- **Universal Instantiation (UI):** If ∀x P(x) is true, we can replace the variable x with any ground term t (a term without variables). For example, from ∀x King(x) ⇒ Person(x), we can infer King(John) ⇒ Person(John).
- **Existential Instantiation (EI):** If ∃x P(x) is true, we can replace x with a new constant symbol (called a Skolem constant) that does not appear anywhere else in our knowledge base. For example, from ∃x Crown(x), we can infer Crown(C1), where C1 is a unique label for that crown.

Unification

Unification is the process of finding a substitution (binding variables to terms) that makes two different logical expressions identical. It is represented as the function Unify(p, q) = theta.

For example, if we have two expressions, Knows(John, x) and Knows(John, Jane), we can unify them by substituting Jane for x:

Unify(Knows(John, x) , Knows(John, Jane)) = { x / Jane }

Resolution in First-Order Logic

Just like in propositional logic, resolution in FOL is the basis of modern inference engines. However, to resolve clauses in FOL, we must perform Skolemization (removing existential quantifiers by replacing variables with Skolem constants/functions) and rename variables to avoid conflicts before applying the resolution rule.

Unit Summary

This unit focused on Knowledge Representation using Logical Formalisms. The key takeaways are summarized below.

- Logical formalisms represent facts as formal sentences, enabling rigorous knowledge-based reasoning.
- Knowledge-based agents manipulate a Knowledge Base (KB) using TELL (assert facts) and ASK (query truth) operations.
- Syntax is the grammar of the logical system; Semantics is the meaning, truth, and models. $KB \models \alpha$ means KB entails α.
- Propositional logic uses symbols (P, Q) and five connectives ($\neg$, $\wedge$, $\vee$, $\Rightarrow$, $\Leftrightarrow$) to build well-formed formulas.
- Inference rules like Modus Ponens, Modus Tollens, and Resolution derive new facts from existing formulas.
- First-Order Logic (FOL) adds expressiveness by representing objects, relations, functions, and variables.
- FOL uses Universal ($\forall$) and Existential ($\exists$) quantifiers to speak about groups of objects.
- Implication ($\Rightarrow$) is paired with $\forall$, and Conjunction ($\wedge$) is paired with $\exists$ to avoid semantic pitfalls.
- Inference in FOL relies on Universal/Existential Instantiation, Unification, and First-Order Resolution.